Proceedings of the IASS-SLTE 2014 Symposium
"Shells, Membranes and Spatial Structures: Footprints"
15 to 19 September 2014, Brasilia, Brazil
Reyolando M.L.R.F. BRASIL and Ruy M.O. PAULETTI (eds.)

Research on Wind-Induced Response Behavior of Monolayer cable net

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Abstract

Based on the random vibration theory, the frequency domain calculation method is adopted to analyze the wind-induced response of monolayer cable net. Some parameters crucial to the response of wind vibration are highlighted, including different algorithm of frequency domain calculation and the selected spatial coherence function. Three analysis patterns of monolayer cable net were established to analyze effect of parameters. The results shown that the coupling effect between modes is significant; the selected coherence function has a great impact on the computation, the coherence function related to wind frequency is more reasonable.

Keywords: cable net, frequency domain calculation, cross part of mode, coherent function

1. Introduction

Tension structure is the wind-sensitive structure due to the light quality and flexible property. Therefore, wind load is the primary control load during the design process. But there is less study on the wind-induced response research of this kind of structure at home and abroad, due to its complexity characteristics in the wind load and structure and lack of understanding of performance in wind exciting force [1].

For the monolayer cable net structure, the existing research shows that the pulsation of the cable net under the average wind load can be considered as linear vibration. Therefore, the theory of frequency domain method can be used for structural wind vibration response analysis. In the present study, the frequency domain calculation method of wind-induced response mainly adopts SRSS method (square root of the sum of the squares method). This method ignores the coupling effective and suitable for the structure with dispersing natural frequency, e.g. high-rise structure. The dynamic property of tension structure is different with the high-rise structure with a closely natural frequency [2]. Therefore, it is needed to be taken into consideration that whether the SRSS method is suitable for the tension structure. Meanwhile, the theory of frequency domain method adopted the spatial coherence function to consider the spatial correlation of wind [3,4]. There are a coherent various function model, but still lack detailed research to study the impact of it.

In order to analyse the above problems, SRSS and CQC methods are adopted to analyze the wind-induced response of monolayer cable net. The effects of cross part of mode and coherent function are discussed.

2. Theory of frequency domain method

Based on the thoery of radom vibration，the dynamic equation of structural under stationary radom fluctuating wind is:

$$\boldsymbol{M}\ddot{\boldsymbol{y}}+\boldsymbol{C}\dot{\boldsymbol{y}}+\boldsymbol{K}\boldsymbol{y}=\boldsymbol{P}\left(\boldsymbol{t}\right) \tag{1}$$

Where the $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ is the matrix of mass, stiffness and damp,respectively. The $\ddot{\boldsymbol{y}}$, $\dot{\boldsymbol{y}}$ and $\boldsymbol{y}$ is the displacement, velocity and accelerate velocity, $\boldsymbol{P}\left(\boldsymbol{t}\right)$ is the force of fluctuating wind。

Using mode analysis method to decouple the equation (1) and the dynamic equation under generalized coordinater system can be obtained:

$$\ddot{q}_j(t)+2\zeta_j w_j \dot{q}(t)+w_j^2 q(t)=f_j(t) \tag{2}$$

Where the q_j, w_j and ζ_j is is the generalized coordinated, natrual frequency and damp ratio of mode j, respectively. $f_j(t)$ is the generalized force of fluctuating wind of mode j.

With using the mode superposition method, the power spectrum of generalized displacement $\boldsymbol{S}_{qq}(\boldsymbol{w})$ can be caculatied:

$$\boldsymbol{S}_{qq}(\boldsymbol{w})=\boldsymbol{H}^{*}\boldsymbol{S}_{ff}(\boldsymbol{w})\boldsymbol{H} \tag{3}$$

Where, $\boldsymbol{H}$ is the matrix of frequency response domain, as $\begin{bmatrix} H_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & H_n \end{bmatrix}$, $\boldsymbol{S}_{ff}(\boldsymbol{w})$ is the matrix of generalized force. The matrix of generalized froce $\boldsymbol{S}_{ff}(\boldsymbol{w})$ can be obtained：

$$\boldsymbol{S}_{ff}(\boldsymbol{w})=\boldsymbol{H}^{*}\boldsymbol{\varphi}^{T}\boldsymbol{S}_{pp}(\boldsymbol{w})\boldsymbol{\varphi}\boldsymbol{H} \tag{4}$$

Where, $\boldsymbol{\varphi}$ is the matrix of mode of vibration， $\boldsymbol{S}_{pp}(\boldsymbol{w})$ is the matrix of loads.

The $\boldsymbol{S}_{pp}(\boldsymbol{w})$ is the matrix of loads and can be expressed by the matrix product of coherent function and the power spectrum of fluctuating wind. The equation is:

$$S_{p_m p_n}(w)=coh(r,w)\sqrt{S_{p_m}(w)S_{p_n}(w)} \tag{5}$$

Where, the $coh(r,w)$ is coheret fucntion, r is the distance of two point, $S_{p_w}(w)$ and $S_{p_q}(w)$ is the power spectrum of fluctuaning of point m and n.

Then, the power spectrum of structral response $\boldsymbol{S}_{yy}(\boldsymbol{w})$ is:

$$\boldsymbol{S}_{yy}(\boldsymbol{w})=\boldsymbol{\varphi}\boldsymbol{S}_{qq}(\boldsymbol{w})\boldsymbol{\varphi}^{T} \tag{6}$$

Now, the root mean square error (RMSE) of each point can be getted by the flowing function:

$$\left\{\sigma_y^2\right\}=\mathrm{diag}\left(\int_0^\infty \boldsymbol{S}_{yy}(\boldsymbol{w})dw\right)=\int_0^\infty \sum_{j=1}^{m}\sum_{k=1}^{m}\varphi_{ji}\varphi_{ki}H_j^{*}(iw)H_k(iw)S_{f_j f_k}(n)dn \tag{7}$$

This equation contains all the cross part of mode, but also lack of efficiency in computation. The Chinese design code [5] is mainly used square root of the sum of the squares (SRSS).

$$\sigma_{yi}^2=\sum_{j=1}^{n}\varphi_{ji}^2\int_0^\infty S_{q_j q_j}(n)dn=\sum_{j=1}^{n}\varphi_{ji}^2\int_0^\infty \left|H(in)\right|^2 S_{f_j f_j}(n)dn \tag{8}$$

3. Issues existing in frequency domain method

3.1. Coupling effect of cross part of mode

Compare to the equation (7), the equation (8) ignore the cross part on different modal. For the high-rise building with discrete modal, the equation (8) can have a good accuracy. But the dynamic property of tension structure is different with the high-rise building. How to select the number of computation modes and consider effect the cross part of mode is key issues in analysis the wind-induced response of this structure. For this reason, two calculation methods are adopted to study the effect of modal coupling which will be described in the section below.

3.2. Coherent function

The spatial correlation of wind is the phenomena that the peak point of fluctuating wind speed on different point won't appear at the same moment. The greater the distance, the less likely it was that the peak point appear on different point [6]. At present, scholars at home and abroad have proposed different coherence functions to consider the spatial correlation. The coherence functions can be classified as two different categories. One consider the frequency effect, the other is not. Equation (9) is the coherent function proposed by Shinotami, which cosider the spatial correlation of vertical and horizontal and ingore the effect of frequency of wind. The Davenport coherent function consider the frequency of wind and spatial correlation (Equation (10)). But the parameters c_x and c_z are greatly effect by the environmental. However, only few studies on how the coherent function impact the results of frequency domain method. For the propuse to study the effect of different coherent fucntion, the Shiotami coherent function and Davenport coherent function are adopted in this paper.

Shiotami coherent function：

$$coh\_1 = \exp\left\{-\left[\left(x_m - x_n\right)^2 \big/ L_x^2 + \left(z_m - z_n\right)^2 \big/ L_z^2\right]^{\frac{1}{2}}\right\} \tag{9}$$

Where, x_m and x_n is the horizontal ordinate of node m and node n, respectively. z_m and z_n is the vertical ordinate of node m and node n. L_x and L_z is the correlation coefficien of vertical and crossange.

Davenport coherent function：

$$coh\_2 = \exp\left\{-w\left[c_x^2\left(x_m - x_n\right)^2 + c_z^2\left(z_m - z_n\right)^2\right]^{\frac{1}{2}} \Big/ \bar{v}_{10}\right\} \tag{10}$$

Where, w is circular frequency, $\bar{v}_{10}$ is the average speed at 10m height, x_m and x_n is the horizontal ordinate of node m and node n, respectively. z_m and z_n is the vertical ordinate of node m and node n, respectively. c_x and c_z is the correlation coefficien of vertical and crossange. Reference [7] recommed parameters in euqation(9) can be: $L_x = 50$, $L_z = 60$. Parameters in equation (10): $c_x = 16$, $c_z = 10$.

4. Numerical example

The height of curtain is 21m and width 18m. The glass is the double tempered laminated glass. The grid of curtrain is 14 columns and 16 rows. The size of grid is 1.5m×1.5m. The boundary condition is fixed. The diameters of vertical cable is 36mm and horizontal cable is 30mm. The elasticity modulus is $1.35E+05\,N/mm^2$. The basic win oressure is $0.45\,kN/m^2$. In order to analyze the effect of different caculation and different spatial coherent function, three case shown in table 1 are studied.

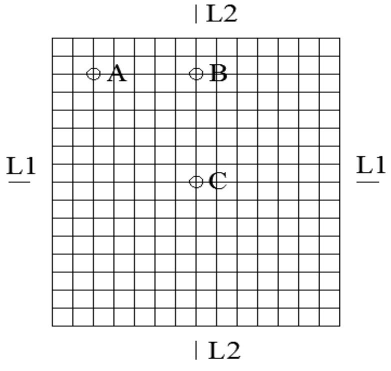


Figure 1: Monolayer cable net

Table 1 Analysis pattern

Analysis pattern	Case 1	Case 2	Case 3
Calculation method	SRSS	CQC	CQC
Coherent function	Davenport	Davenport	Shiotami

5. Analysis of the calculation results

5.1. Effect of cross part of mode on dynamic displacement

The parts of first 200 model natural frequencies are shown in table 2. The table 2 is shown that the natural frequencies are closely.

For the purpose of analysis the effect of different calculation method, root mean square (RMS) of displacement response of Node A~Node C are plotted. From the figure, the following conclusions can be conclued: (1) the cross part of mode has a significant effect on the results and has different effect on each point. For the Node A and Node B, the results has increased by 24.23% and 7.71%. But the results of Node C decreased by 9.01%; (2) the effect of cross part of mode keep stabilization with the number of computing modes growing. From the conclusion we concluded above, the cross part of mode should be considered in caculating the wind-induced response of monolayer cable net.

Table 2 Natural frequency of cable net

Mode	1	2	3	4	5
Frequency/Hz	1.295	1.832	2.226	2.465	2.575
Mode	10	50	100	150	200
Frequency/Hz	3.61	6.88	8.706	9.888	11.001

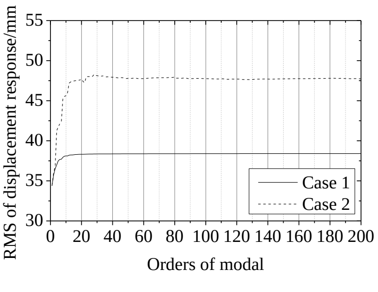


Figure 2: Node A

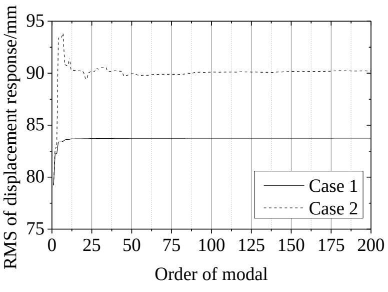


Figure 3: Node B

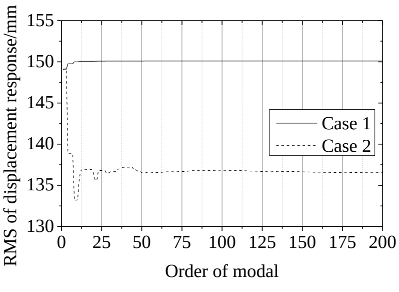
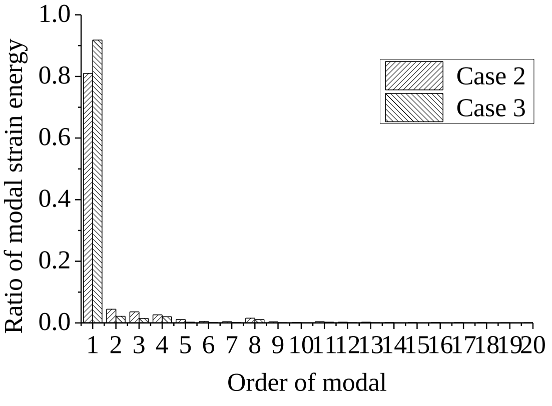


Figure 4: Node C

5.2. Selection of vibration mode

At present, the method to consider the contribution of each mode is using the method proposed by reference [8]. Fig.5 showed the distribution of the ratio of modal strain energy of first twenty modes. The result indicates the first mode occupy a greater proportion. With the increasing number of vibration mode, the modal strain energy of each mode is decreasing. The jump phenomenon isn’t appear [9]. Fig.6 showed the distribution of ratio accumulative modal strain energy of first 200 modes.The modal energy is stabilizing after 50 modes and the ratio of modal accumulative modal strain energy up to 95%. This phenomenon indicates the previous mode contributes the main contribution. Refer to the Fig.2 ~ Fig.4, when the accumulative of modal strain energy up to the 95%. The modal coupling effects are approximately constant. It suggests that the selection of vibration mode can be first 50 modes.



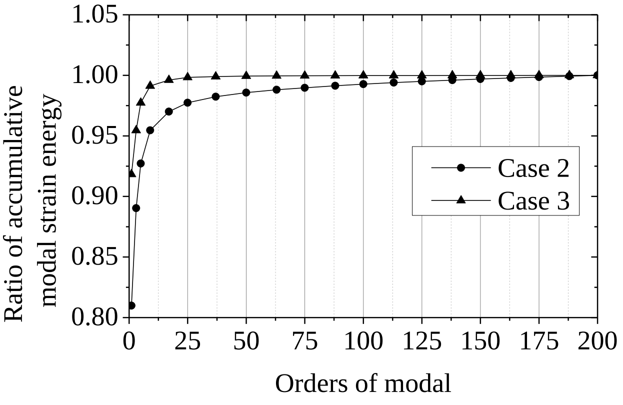
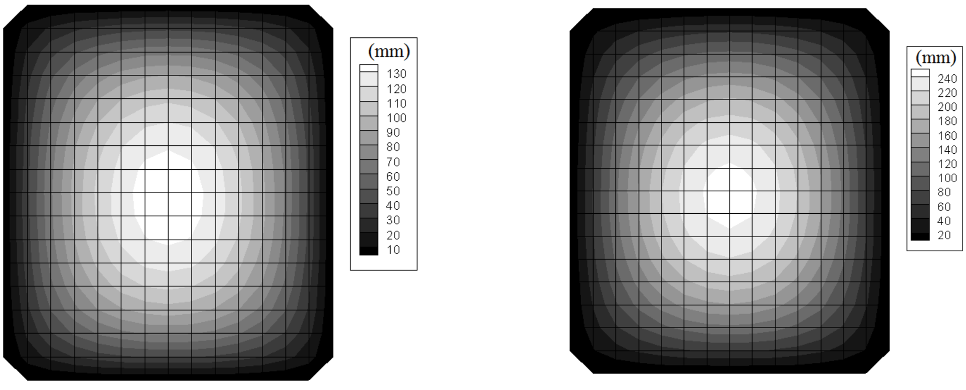


Figure 5: Ratio of modal strain energy

Figure 6: Ratio of accumulative modal strain energy

5.3. Effect of coherent function on dynamic displacement

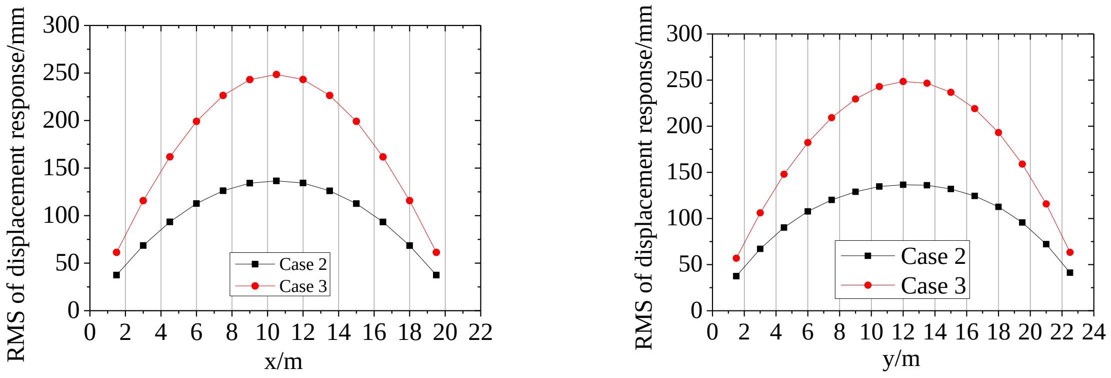
The Fig.7 show that the largest response of displacement appears in the centre of the cable net. The results indicate different coherent function has a significant effect on the calculation results. The maximum value is 0.229mm when the Shiotami coherent function is selected. But the maximum value is 0.125mm when choose the Davenport coherent function. In combination with the reference [10] and [11], it's more reasonable to use the Davenport coherent function which considered the frequency of wind.



(a) Davenport coherent function

(b) Shiotami coherent function

Figure 7: RMS of vertical displacement with different coherent function



(a) L1

(b) L2

Figure 8: RMS of vertical displacement at different axis

6. Conclusion

This paper adopted SRSS and CQC method to analyze the wind-induced vibration of monolayer cable net. The effect of cross part of mode and coherent function is analyzed emphatically.

(1) Due to the natural frequency of monolay cable net is closely. The effect of cross part of mode is significant. For the node point at different location, the effect can be positive or negative. So, it is recommended that the CQC method should be adopted to calculate the wind-induced response of monolayer cable net.

(2) The phenomena of the jump are not appeared in the results of the ratio of modal strain energy. The top 50 model has the significant effect on the wind-induced response.

(3) The results of wind-induced response with using different coherent function are significant differences.

Acknowledgement

The research described in this paper was financially supported by the National "Twelfth Five-Year" Plan for Science & Technology Support (No. 2011BAJ08B10)

References

[1] Wu Yue, Feng Ruo-qiang, Shen shi-zhao. Wind-inducedresponse analysis and practical. *Chinese Journal of Computational Mechanics*, 2007, 24(5): 633-637. (in Chinese)

[2] Zhou Xuan-yi, Gu ming, Li Ya-ming, et al. Wind-induced responses analysis for a large-scale museum structure. *Journal of vibration and shock*, 2008, 2: 004. (in Chinese)

[3] Simiu E, Scanlan R H. *Wind effects on structure-An introduction to wind engineering*. Shanghai: Tongji University press. 1992. (in Chinese)

[4] Davenport A G. The relationship of wind structure to wind loading. *In proceeding of the symposium on wind effects on building and structres*, London, 1966, 1, 54-102.

[5] GB50009-2001, *Load code for the design of buildingstructures*. Beijing: China Architecture Industry Press.2002. (in Chinese)

[6] Huang Ben-cai, Wang Cong-jun. *The principle and application of structural wind resistance analysis*. Shanghai: Tongji University Press, 2001. (in Chinese)

[7] Zhang Xiang-ting. *Structural Wind Engineering: Theory ·Standard ·Practice*. Beijing: China Building Industry Press, 2006. (in Chinese)

[8] Nakayama M, Sasaki Y, Masuda K, et al. An efficient method for selection of vibration modes contributory to wind response on dome-like roofs. *Journal of Wind Engineering and Industrial Aerodynamics*, 1998, 73(1): 31-43.

[9] He Yan-li, Dong Shi-lin ,Gong Jing-hai. Wind-induced response of spatial structures with mode compensation in frequency domain. *Engineering Mechanics*, 2002, 19(4): 1~6. (in Chinese)

[10] Wu Lili, Wang Yuan-qing, Shi Yongjiu. Research on nonlinear characteristics of wind-induced responses ofmonolayer cable net. *Journal of Vibration and Shock*, 2007,26(4): 30~34. (in Chinese)

[11] Holmes J D. *Wind loading of structures*. Taylor & Francis, 2001.