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Revised finite element formulation for membrane creep analysis

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Abstract

Membranes, such as PVC, PTFE and ETFE, are widely used in spatial structures, and some of them exhibit viscoelastic-plastic property in long-term tension condition. Meanwhile, finite element method (FEM) has been a significant tool for structural analysis. This paper focuses on finite element formulation for viscoelastic-plastic modelling and its application for membrane creep analysis. On the basis of Schapery's nonlinear viscoelastic modelling, Kennedy presented finite element formulation for creep analysis. This paper developed Kennedy's work to consider viscoplastic component, which would not recover in given time. Revised finite element formulation for the viscoelastic-plastic modelling was presented, which requiring merely quantities at current time step and previous time step, rather than quantities at whole time scale. This formulation considered both viscoelastic creep deformation and viscoplastic creep deformation, and had been programmed in FORTRAN. Two examples were used to verify this formulation. One example was a rectangle specimen under uniaxial tension. In 10 hours creep time, total creep displacement was 1.79 mm in theoretical and 1.82 in numerical simulation. Simulation creep curve also embraced good agreement comparing with theoretical result during creep process. The other example was a triangle ETFE cushion. This cushion was tested in 12 hours for creep analysis. Inner pressure was 825 Pa. Creep properties at four stresses were offered and at other stresses were determined by linear interpolation. Creep displacements at the center points of the upper and lower foils were both measured by laser sensors. The maximum creep displacement was about 3.0 mm in 12 hours creep time. Numerical simulation was done and its result was compared with test data, which showing good agreement. Therefore, the revised finite element formulation is valid in membrane structures for creep analysis.

Keywords: finite element formulation, viscoelastic-plastic modelling, creep analysis, membrane structures, numerical simulation.

1. Introduction

Membranes, such as PVC, PTFE and ETFE, are novel structural materials being used in spatial buildings. As kinds of composite materials or polymers, these materials exhibit viscoelastic or viscoelastic-plastic property in their serving life, which might cause stress relaxation or creep deformation. Meanwhile, during recent decades, finite element method (FEM) has been successfully applied in structural analysis and design. It is a significant tool for researchers and engineers to analyze complex structural response under various loading condition, of course, including creep behavior of membranes.

On the basis of Schapery's nonlinear constitutive equation in single integral form [1,2], several finite element formulations have been proposed. Kennedy presented finite element analysis for viscoelastic response of laminated composites, for both shell element and three-dimensional element [3,4]. Zocher developed a thermo viscoelastic finite element formulation for linear uncoupled thermo viscoelastic stress analysis and matrix-cracked laminates [5]. To discretize the integral operators, Chazal proposed a mathematical approach for the solution of linear, non-aging viscoelastic materials undergoing mechanical deformation [6]. Moreover, Chung developed viscoelastic/rate-sensitive-plastic constitutive law for fiber reinforced composites and applied it in both loading process and creep analysis [7,8].

However, while membranes in structure exhibit viscoelastic and viscoplastic behavior in long-term tension condition [9,10], prior researches focused merely on nonlinear viscoelastic constitutive equation of composites, in which viscoelastic and viscoplastic creep deformations have not been distinguished. Therefore, this paper revised finite element formulation for creep analysis of membrane, and developed the formulation to consider viscoplastic component.

2. Constitutive equations

Schapery's nonlinear constitutive modelling for creep analysis in uniaxial loading condition could be expressed as Eq. (1).

$$\varepsilon(t) = g_0(t)D_0\sigma(t) + g_1(t)\int_0^t D_c(\psi(t)-\psi(\tau))\frac{\partial}{\partial\tau}(g_2(\tau)\sigma(\tau))\mathrm{d}\tau + \Theta(t) \tag{1}$$

The reduced time in Eq. (1) is defined by Eq. (2).

$$\psi(t) = \int_0^t \frac{\mathrm{d}u}{a} \text{ and } \psi(\tau) = \int_0^\tau \frac{\mathrm{d}u}{a} \tag{2}$$

where $\varepsilon(t)$ is the strain being dependent on time, $\sigma(t)$ is the stress, $g_0(t)$, $g_1(t)$, $g_2(\tau)$ and a are nonlinear coefficients relating to stress, temperature and strain rate, D_0 is initial compliance, D_c is transient compliance, $\Theta(t)$ is the thermal strain, which is not discussed in this paper.

To distinguish viscoelastic and viscoplastic creep deformation on the basis of creep and recovery tests, viscoelastic-plastic modelling is adopted to represent transient compliance D_c expression, as shown in Eq. (3).

$$D_c(\psi(t)-\psi(\tau)) = \sum_{r=1}^{N} D_r\left[1-e^{-\lambda_r(\psi(t)-\psi(\tau))}\right] + D_f(\psi^k(t)-\psi^k(\tau)) \tag{3}$$

where D_r and λ_r are creep coefficients in viscoelastic component which would recover in given time, D_f and k are creep coefficients in viscoplastic component.

Then, substituting Eq. (3) into Eq. (1), total strain $\varepsilon_i(t)$ in time step t_n could be rewritten in hyper matrix form, as shown in Eq. (4).

$$\varepsilon_{ij}(t_n) = \xi_{0r}(t_n) - \xi_r(t_n) + \xi_f(t_n) \tag{4}$$

where

$$\xi_{0r}(t_n) = \sum_{k=1}^{3}\sum_{l=1}^{3}\left[g_{0ijkl}(t_n)D_{0ijkl}\sigma_{kl}(t_n) + g_{1ijkl}(t_n)g_{2ijkl}(t_n)\sum_{r=1}^{N}D_{rijkl}\sigma_{kl}(t_n)\right] \tag{5-a}$$

$$\xi_r(t_n) = \sum_{k=1}^{3}\sum_{l=1}^{3}\left[g_{1ijkl}(t_n)\int_0^{t_n}\sum_{r=1}^{N}D_{rijkl}e^{-\lambda_{rijkl}(\psi_{ijkl}(t_n)-\psi_{ijkl}(\tau))}\frac{\partial}{\partial\tau}(g_{2ijkl}(\tau)\sigma_{kl}(\tau))\mathrm{d}\tau\right] \tag{5-b}$$

$$\xi_f(t_n) = \sum_{k=1}^{3}\sum_{l=1}^{3}\left[g_{1ijkl}(t_n)D_{fijkl}\int_0^{t_n}(\psi_{ijkl}{}^k(t_n)-\psi_{ijkl}{}^k(\tau))\frac{\partial}{\partial\tau}(g_{2ijkl}(\tau)\sigma_{kl}(\tau))\mathrm{d}\tau\right] \tag{5-c}$$

As proposed by Kennedy, Eq. (5-b) is given in Eq. (6). This equation only requires quantities at current time step t_n and previous time step t_{n-1}, rather than quantities at the whole time scale, which save computational memory in numerical simulation.

$$\xi_r(t_n) = \sum_{k=1}^{3}\sum_{l=1}^{3}g_{1ijkl}(t_n)\left[e^{-\lambda_{rijkl}\psi_{ijkl}(\Delta t_n)}\xi_r(t_{n-1}) + \sum_{r=1}^{N}D_{rijkl}(1-e^{-\lambda_{rijkl}\psi_{ijkl}(\Delta t_n)})\frac{g_{2ijkl}(t_n)\sigma_{kl}(t_n)-g_{2ijkl}(t_{n-1})\sigma_{kl}(t_{n-1})}{\lambda_{rijkl}\psi_{ijkl}(\Delta t_n)}\right] \tag{6}$$

When it turns to the viscoplastic component (5-c), finite element formulation is conducted as following. Similar as Eq. (6), this component is given in Eq. (7), requiring quantities at current time step t_n and previous time step t_{n-1}. In order to simplify the formulation, g_{1ijkl}, g_{2ijkl} are assumed not changing with time, namely $g_{1ijkl}(t_n)= g_{1ijkl}(t_{n-1})$, $g_{2ijkl}(t_n)= g_{2ijkl}(t_{n-1})$. Also, $\psi(t_n)$ is assumed equaling to $\psi(t_{n-1})+ \psi(\Delta t)$.

$$
\begin{aligned}
\xi_f(t_{\mathrm{n}}) &= \sum_{k=1}^{3}\sum_{l=1}^{3}\Bigg\{ g_{1ijkl}(t_{\mathrm{n}})D_{fijkl}\Bigg[\int_0^{t_{\mathrm{n}\text{-}1}}((\psi_{ijkl}(t_{\mathrm{n}\text{-}1})+\psi_{ijkl}(\Delta t))^k-\psi_{ijkl}{}^k(\tau))\frac{\partial}{\partial\tau}(g_{2ijkl}(\tau)\sigma_{kl}(\tau))\mathrm{d}\tau \\
&\quad +\int_{t_{\mathrm{n}}-1}^{t_{\mathrm{n}}}(\psi_{ijkl}{}^k(t_{\mathrm{n}})-\psi_{ijkl}{}^k(\tau))\frac{\partial}{\partial\tau}(g_{2ijkl}(\tau)\sigma_{kl}(\tau))\mathrm{d}\tau\Bigg]\Bigg\} \\
&= \xi_f(t_{\mathrm{n}\text{-}1})+\sum_{k=1}^{3}\sum_{l=1}^{3}\Bigg\{ g_{1ijkl}(t_{\mathrm{n}})D_{fijkl}\Bigg[\int_0^{t_{\mathrm{n}\text{-}1}}k\psi_{ijkl}{}^{k-1}(t_{\mathrm{n}\text{-}1})\psi_{ijkl}(\Delta t)\frac{\partial}{\partial\tau}(g_{2ijkl}(\tau)\sigma_{kl}(\tau))\mathrm{d}\tau \\
&\quad +\int_{t_{\mathrm{n}}-1}^{t_{\mathrm{n}}}(\psi_{ijkl}{}^k(t_{\mathrm{n}})-\psi_{ijkl}{}^k(\tau))\frac{\partial}{\partial\tau}(g_{2ijkl}(\tau)\sigma_{kl}(\tau))\mathrm{d}\tau\Bigg]\Bigg\} \\
&= \xi_f(t_{\mathrm{n}\text{-}1})+\sum_{k=1}^{3}\sum_{l=1}^{3}\Big\{ g_{1ijkl}(t_{\mathrm{n}})D_{fijkl}\Big[k\psi_{ijkl}{}^{k-1}(t_{\mathrm{n}\text{-}1})\psi_{ijkl}(\Delta t)g_{2ijkl}(t_{\mathrm{n}\text{-}1})\sigma_{kl}(t_{\mathrm{n}\text{-}1}) \\
&\quad +\psi_{ijkl}{}^k(t_{\mathrm{n}})(g_{2ijkl}(t_{\mathrm{n}})\sigma_{kl}(t_{\mathrm{n}})-g_{2ijkl}(t_{\mathrm{n}\text{-}1})\sigma_{kl}(t_{\mathrm{n}\text{-}1}))-\int_{t_{\mathrm{n}}-1}^{t_{\mathrm{n}}}\psi_{ijkl}{}^k(\tau)\frac{\partial}{\partial\tau}(g_{2ijkl}(\tau)\sigma_{kl}(\tau))\mathrm{d}\tau\Big]\Big\} \\
&= \xi_f(t_{\mathrm{n}\text{-}1})+\sum_{k=1}^{3}\sum_{l=1}^{3}\Big\{ g_{1ijkl}(t_{\mathrm{n}})D_{fijkl}\Big[k\psi_{ijkl}{}^{k-1}(t_{\mathrm{n}\text{-}1})\psi_{ijkl}(\Delta t)g_{2ijkl}(t_{\mathrm{n}\text{-}1})\sigma_{kl}(t_{\mathrm{n}\text{-}1}) \\
&\quad +\psi_{ijkl}{}^k(t_{\mathrm{n}})(g_{2ijkl}(t_{\mathrm{n}})\sigma_{kl}(t_{\mathrm{n}})-g_{2ijkl}(t_{\mathrm{n}\text{-}1})\sigma_{kl}(t_{\mathrm{n}\text{-}1})-\psi_{ijkl}{}^k(t_{\mathrm{n}})g_{2ijkl}(t_{\mathrm{n}})\sigma_{kl}(t_{\mathrm{n}})+\psi_{ijkl}{}^k(t_{\mathrm{n}\text{-}1})g_{2ijkl}(t_{\mathrm{n}\text{-}1})\sigma_{kl}(t_{\mathrm{n}\text{-}1}) \\
&\quad +\frac{1}{2}(\psi_{ijkl}{}^k(t_{\mathrm{n}})-\psi_{ijkl}{}^k(t_{\mathrm{n}\text{-}1}))(g_{2ijkl}(t_{\mathrm{n}})\sigma_{kl}(t_{\mathrm{n}})+g_{2ijkl}(t_{\mathrm{n}\text{-}1})\sigma_{kl}(t_{\mathrm{n}\text{-}1}))\Big]\Big\} \qquad (7) \\
&= \xi_f(t_{\mathrm{n}\text{-}1})+\sum_{k=1}^{3}\sum_{l=1}^{3}\Bigg\{ g_{1ijkl}(t_{\mathrm{n}})D_{fijkl}\Bigg[\frac{1}{2}(\psi_{ijkl}{}^k(t_{\mathrm{n}})-\psi_{ijkl}{}^k(t_{\mathrm{n}\text{-}1}))(g_{2ijkl}(t_{\mathrm{n}})\sigma_{kl}(t_{\mathrm{n}})+g_{2ijkl}(t_{\mathrm{n}\text{-}1})\sigma_{kl}(t_{\mathrm{n}\text{-}1}))\Bigg]\Bigg\}
\end{aligned}
$$

Hence, by substituting Eq. (6), Eq. (7) into Eq. (4), total strain for the viscoelastic-plastic modelling is rewritten in Eq. (8), which could be conveniently used in finite element analysis.

$$
\begin{aligned}
\varepsilon_{ij}(t_{\mathrm{n}}) &= \sum_{k=1}^{3}\sum_{l=1}^{3}\Bigg\{\Bigg[g_{0ijkl}(t_{\mathrm{n}})D_{0ijkl}+g_{1ijkl}(t_{\mathrm{n}})g_{2ijkl}(t_{\mathrm{n}})\sum_{r=1}^{N}D_{rijkl}-g_{1ijkl}(t_{\mathrm{n}})g_{2ijkl}(t_{\mathrm{n}})\sum_{r=1}^{N}D_{rijkl}\frac{1-e^{-\lambda_{rijkl}\psi_{ijkl}(\Delta t_{\mathrm{n}})}}{\lambda_{rijkl}\psi_{ijkl}(\Delta t_{\mathrm{n}})} \\
&\quad +g_{1ijkl}(t_{\mathrm{n}})g_{2ijkl}(t_{\mathrm{n}})D_{fijkl}\frac{1}{2}(\psi_{ijkl}{}^k(t_{\mathrm{n}})-\psi_{ijkl}{}^k(t_{\mathrm{n}\text{-}1}))\Bigg]\sigma_{kl}(t_{\mathrm{n}}) \qquad (8) \\
&\quad +\Bigg[-g_{1ijkl}(t_{\mathrm{n}})e^{-\lambda_{rijkl}\psi_{ijkl}(\Delta t_{\mathrm{n}})}\xi_r(t_{\mathrm{n}\text{-}1})+g_{1ijkl}(t_{\mathrm{n}})g_{2ijkl}(t_{\mathrm{n}\text{-}1})\sum_{r=1}^{N}D_{rijkl}\frac{1-e^{-\lambda_{rijkl}\psi_{ijkl}(\Delta t_{\mathrm{n}})}}{\lambda_{rijkl}\psi_{ijkl}(\Delta t_{\mathrm{n}})}\sigma_{kl}(t_{\mathrm{n}\text{-}1}) \\
&\quad +\xi_f(t_{\mathrm{n}\text{-}1})+g_{1ijkl}(t_{\mathrm{n}})g_{2ijkl}(t_{\mathrm{n}})D_{fijkl}\frac{1}{2}(\psi_{ijkl}{}^k(t_{\mathrm{n}})-\psi_{ijkl}{}^k(t_{\mathrm{n}\text{-}1}))\sigma_{kl}(t_{\mathrm{n}\text{-}1})\Bigg]\Bigg\}
\end{aligned}
$$

3. Finite element analysis

Basing on the principle of virtual work, incremental equilibrium equations for finite element analysis are shown in Eq. (9).

$$\int_{Ve}\sigma_{ij}(t_n)\delta\varepsilon_{ij}(t_n)\mathrm{d}V=\int_{Ve}f_i^v(t_n)\delta u_i(t_n)\mathrm{d}V+\int_{Se}T_i^s(t_n)\delta u_i(t_n)\mathrm{d}S \qquad (9)$$

where

$$\sigma_{ij}(t_n)=\sigma_{ij}(t_{n-1})+\Delta\sigma_{ij},\varepsilon_{ij}(t_n)=\varepsilon_{ij}(t_{n-1})+\Delta\varepsilon_{ij},u_i(t_n)=u_{ij}(t_{n-1})+\Delta u_{ij}$$

and

$$\delta\varepsilon_{ij}(t_n)=\delta(\Delta\varepsilon_{ij}),\delta u_{ij}(t_n)=\delta(\Delta u_{ij})$$

f_i^v and T_i^s in Eq. (9) represent body load and surface load, respectively.

Then, Eq. (9) could be rewritten in Eq. (10).

$$\int_{Ve}\Delta\sigma_{ij}\delta(\Delta\varepsilon_{ij})\mathrm{d}V=\int_{Ve}f_i^v(t_n)\delta(\Delta u_i)\mathrm{d}V+\int_{Se}T_i^v(t_n)\delta(\Delta u_i)\mathrm{d}S-\int_{Ve}\sigma_{ij}(t_{n-1})\delta(\Delta\varepsilon_{ij})\mathrm{d}V \qquad (10)$$

According to finite element formulation for the viscoelastic-plastic modelling Eq. (8), stress at current time step t_n have a relation with stress at previous time step t_{n-1}, strain at current time step t_n, and quantities at previous time step t_{n-1}. It could be rearranged as form Eq. (11).

$$\sigma_{ij}(t_n) = S(t_n)^{-1}(\varepsilon_{kl}(t_n) - H(t_n)) \tag{11}$$

where

$$S(t_n) = \sum_{k=1}^{3}\sum_{l=1}^{3}\left[g_{0ijkl}(t_\mathrm{n})D_{0ijkl} + g_{1ijkl}(t_\mathrm{n})g_{2ijkl}(t_\mathrm{n})\sum_{r=1}^{N}D_{rijkl} - g_{1ijkl}(t_\mathrm{n})g_{2ijkl}(t_\mathrm{n})\sum_{r=1}^{N}D_{rijkl}\frac{1-e^{-\lambda_{rijkl}\psi_{ijkl}(\Delta t_\mathrm{n})}}{\lambda_{rijkl}\psi_{ijkl}(\Delta t_\mathrm{n})} \right.$$
$$\left. + g_{1ijkl}(t_\mathrm{n})g_{2ijkl}(t_\mathrm{n})D_{fijkl}\frac{1}{2}(\psi_{ijkl}{}^{k}(t_\mathrm{n}) - \psi_{ijkl}{}^{k}(t_\mathrm{n\text{-}1})) \right]$$

$$H(t_n) = \sum_{k=1}^{3}\sum_{l=1}^{3}\left[-g_{1ijkl}(t_\mathrm{n})e^{-\lambda_{rijkl}\psi_{ijkl}(\Delta t_\mathrm{n})}\xi_r(t_\mathrm{n\text{-}1}) + g_{1ijkl}(t_\mathrm{n})g_{2ijkl}(t_\mathrm{n\text{-}1})\sum_{r=1}^{N}D_{rijkl}\frac{1-e^{-\lambda_{rijkl}\psi_{ijkl}(\Delta t_\mathrm{n})}}{\lambda_{rijkl}\psi_{ijkl}(\Delta t_\mathrm{n})}\sigma_{kl}(t_\mathrm{n\text{-}1}) \right.$$
$$\left. + \xi_f(t_\mathrm{n\text{-}1}) + g_{1ijkl}(t_\mathrm{n})g_{2ijkl}(t_\mathrm{n})D_{fijkl}\frac{1}{2}(\psi_{ijkl}{}^{k}(t_\mathrm{n}) - \psi_{ijkl}{}^{k}(t_\mathrm{n\text{-}1}))\sigma_{kl}(t_\mathrm{n\text{-}1}) \right]$$

To simplify the formulation, we assume

$$\Delta\sigma_{ij} = \sigma_{ij}(t_n) - \sigma_{ij}(t_{n-1}) = S(t_n)^{-1}\Delta\varepsilon_{kl} \tag{12}$$

In this way, finite element formulation for the viscoelastic-plastic modelling is written in Eq. (13).

$$\int_{Ve} S(t_n)^{-1}\Delta\varepsilon_{kl}\delta(\Delta\varepsilon_{ij})\mathrm{d}V = \int_{Ve} f_i^{v}(t_n)\delta(\Delta u_i)\mathrm{d}V + \int_{Se} T_i^{v}(t_n)\delta(\Delta u_i)\mathrm{d}S - \int_{Ve} S(t_{n-1})^{-1}(\sigma_{ij}(t_{n-1}) - H(t_{n-1}))\delta(\Delta\varepsilon_{ij})\mathrm{d}V \tag{13}$$

4. Numerical examples

The revised finite element formulation for viscoelastic-plastic modelling is programmed in FORTRAN and is verified by the following two examples. One is for uniaxial creep analysis and compares with theoretical analysis. The other one is for ETFE cushion creep analysis and compares with test results.

4.1. Example for uniaxial creep analysis

This example is a rectangle specimen under uniaxial tension. The specimen is 100 mm in length and 20 mm in width. Its thickness is 1 mm. The specimen is uniaxial tensioned in 0.04N, and kept constant 0.002 MPa in the creep test. Twenty triangle membrane elements are used to divide this specimen. One end is fixed and the other end is forced, as illustrated in Fig. 1. Total creep time in this example is 36000 seconds (10 hours).

Viscoelastic-plastic modelling adopted for this specimen is shown in Eq. (14).

$$D = D_0 + D_1(1 - e^{-t/\tau_1}) + D_f t^k \tag{14}$$

where, D_0=2.0 MPa^{-1}, D_1=5.0 MPa^{-1}, τ_1=10^4 s, D_f=0.5 MPa^{-1}, k=0.2.

Initial displacement in loading process is 0.4 mm, which contains only elastic deformation. The creep displacement in theoretical analysis is 1.79 mm, in which the Poisson's ratio is not considered for the small creep deformation. In numerical simulation, creep displacement is 1.82 mm. Creep displacement of the loading end of specimen is illustrated in Fig. 2. As shown, simulation result has a good agreement comparing with theoretical one.

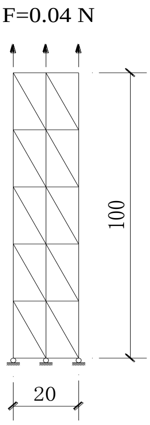

Figure 1. Modelling for uniaxial creep example

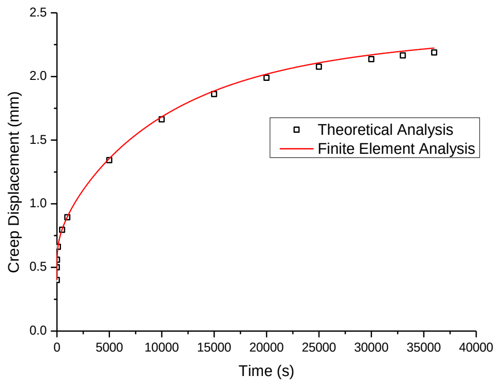

Figure 2. Comparison of theoretical and simulation results

4.2. Example for cushion creep analysis

This example is a double-layer ETFE cushion with 4.0 m in each edge. Creep test of this cushion was performed. The specimen was regular triangle designed by shape finding analysis, in which inner pressure was 450 Pa and stress was 3.56 MPa. The cushion was cut and welded to form its curve shape as form finding result. In the creep test, creep displacement at the center points (T1 and T5) of the upper and lower foils were both measured by laser sensors. Inner pressure during test was 825 Pa. Total creep time in this example is 43200 seconds (12 hours).

The viscoelastic-plastic modelling adopted in this example is shown in Eq. (15), and creep coefficients are listed in Table 1, which were obtained by uniaxial creep tests of ETFE foil using different tensile stress σ.

$$\varepsilon(t)=\left[D_0+\sum_{r=1}^{5}D_r(1-e^{-\frac{t}{10^r}})+D_f t^k\right]\sigma \tag{15}$$

As listed in Table 1, creep property of ETFE foil has a relation with stress. Hence, creep coefficients in each element are all determined according to its stress after loading process. By means of Table 1, creep coefficients are calculated by linear interpolation according to stress.

Table 1 Creep coefficients of ETFE foil

σ	D_0	D_1	D_2	D_3	D_4	D_5	D_f	k
3 MPa	1.228E-3	2.770E-6	4.597E-6	3.145E-5	4.869E-5	4.757E-5	1.635E-7	0.53887
6 MPa	1.296E-3	8.311E-6	5.617E-5	1.466E-4	1.937E-4	1.110E-4	9.752E-7	0.54189
9 MPa	1.378E-3	1.334E-5	1.349E-4	3.667E-4	3.970E-4	2.419E-4	5.319E-7	0.69900
12 MPa	1.431E-3	2.380E-5	2.303E-4	4.588E-4	3.711E-4	4.251E-4	1.554E-5	0.43873

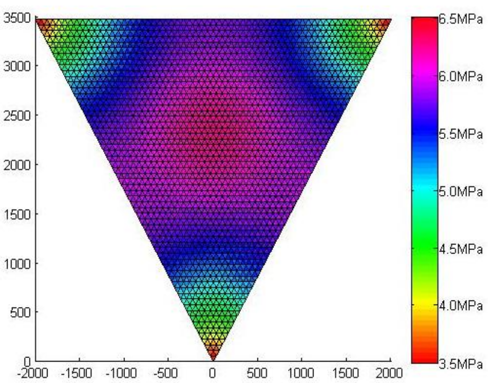

Figure 3. Stress distribution in triangle ETFE cushion at the beginning of creep test

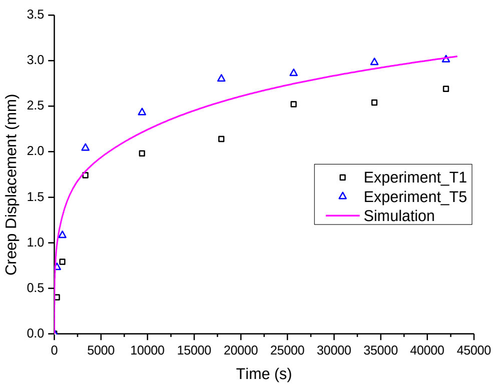

Figure 4. Creep displacement of triangle cushion

Stress distribution of this triangle cushion is numerically analyzed and shown in Fig. 3. Creep coefficients for creep analysis of cushion are given by using Table 1 and the stress distribution in Fig 3.

As recorded by laser sensors in creep test, creep displacements at the center points of this cushion are shown in Fig. 4. In the 12 hours creep time, total creep displacement is about 3.0 mm. Numerical simulation are also performed and illustrated in Fig. 4, the results of which have good agreement comparing with test data.

5. Conclusions

The revised finite element formulation for creep analysis was presented in the paper and examined by two numerical examples.

Total strain equation for the viscoelastic-plastic modelling was proposed firstly. While the nonlinear viscoelastic component was based on Kennedy's work, viscoplastic part was suggested in this paper. This equation required quantities at previous time step, rather than those at whole time scale, to simulate creep strain. Finite element formulation for this viscoelastic-plastic modelling was presented.

The formulation was programmed in FORTRAN and verified by two examples. One example was to simulate a rectangle specimen in uniaxial creep in 10 hours. Simulation result showed good agreement comparing with theoretical analysis. The other one was to simulate the creep test of a triangle ETFE cushion in 12 hours. Creep displacements from numerical simulation have good agreement with those from creep test.

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References

[1] Schapery R.A., On the characterization of nonlinear viscoelastic materials. *Polymer Engineering and Science*, 1969; **9**; 295-310.

[2] Lou Y.C. and Schapery R.A. Viscoelastic characterization of a nonlinear fiber-reinforced plastic. *Journal of Composite Materials*. 1971; **5**; 208-234.

[3] Kennedy T.C., Nonlinear viscoelastic analysis of composite plates and shells. *Composite structures*. 1998; **41**; 265-272.

[4] Kennedy T.C. and Wang M. Three-dimensional, nonlinear viscoelastic analysis of laminated composites. *Journal of composite materials*. 1994; **28**; 902-925.

[5] Zocher M.A., Groves S.E. and Allen D.H., A three-dimensional finite element formulation for thermoviscoelastic orthotropic media. *International Journal for Numerical Methods in Engineering*. 1997; **40**; 2267-2288.

[6] Chazal C. and Pitti R.M., Incremental constitutive formulation for time dependent materials: creep integral approach. *Mechanics of Time-Dependent Materials*. 2011; **15**; 239-253.

[7] Chung K. and Ryou H., Development of viscoelastic/rate-sensitive-plastic constitutive law for fiber-reinforced composites and its applications. Part I: Theory and material characterization. *Composites Science and Technology*. 2009; **69**; 284-291.

[8] Ryou H. and Chung K., Development of viscoelastic/rate-sensitive-plastic constitutive law for fiber-reinforced composites and its applications Part II: Numerical formulation and verification. *Composites Science and Technology*. 2009; **69**; 292-299.

[9] Kawabata M., Viscoplastic Properties of ETFE Film and Structural Behavior of Film Cushion, in *IASS 2007. Shell and Spatial Structures: Structural Architecture-Towards the future looking to the past*, Majowiecki M (ed.), Venice, 2007, 191-192.